Introduction to Fuzzy Logic and Fuzzy Systems

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Objectives

- What Is Fuzzy Logic?
- Fuzzy sets
- Membership function
- Differences between Fuzzy and Probability?
- Fuzzy Inference.
- Why Fuzzy Logic?

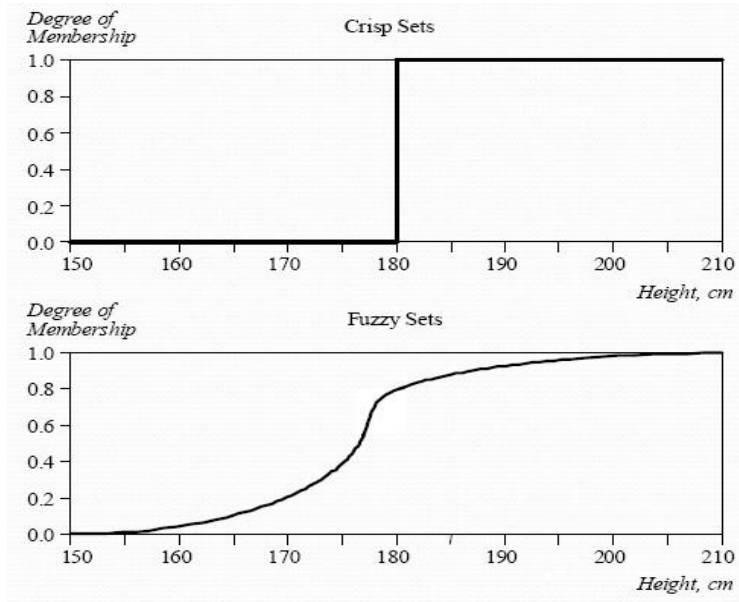
What Is Fuzzy Logic?

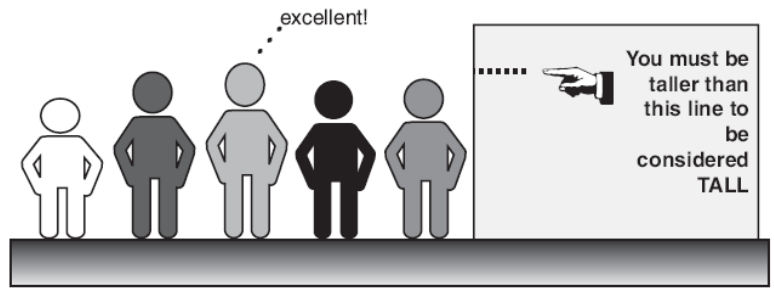
- Theory of fuzzy sets
  - Membership is a matter of degree.
  - Fuzzy sets VS classical set theory.
- Basic foundations of fuzzy sets
  - Fuzzy sets (Zadeh, 1965) , Fuzzy Logic (Zadeh, 1973)
- Fuzzy
  - Reflect how people think
  - Attempts to model our sense of words decision making, and common sense.
  - Mathematical principles for knowledge representation based on degrees of membership rather than on crisp membership of classical binary logic.

Fuzzy sets

- Accept that things can be partly true and partly false to any degree at the same time.
- Crisp and fuzzy sets of 'tall men'

Name	Height, cm	Degree of Membership	
		Crisp	*Fuzzy*
Chris	208	1	1.00
Mark	205	1	1.00
John	198	1	0.98
Tom	181	1	0.82
David	179	0	0.78
Mike	172	0	0.24
Bob	167	0	0.15
Steven	158	0	0.06
Bill	155	0	0.01
Peter	152	0	0.00





Membership function

- Crisp set representation
  - Characteristic function

$$f_A(x): X \to 0,1$$

$$f_A(x) = \begin{cases} 1, \text{if } x \in A \\ 0, \text{if } x \notin A \end{cases}$$

- Fuzzy set representation
  - Membership function

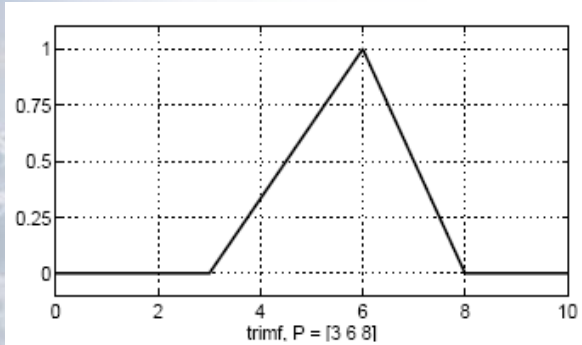
$$\mu_A(x): X \to [0,1]$$

$$\mu_A(x) = 1 \text{ if } x \text{ is totally in } A;$$

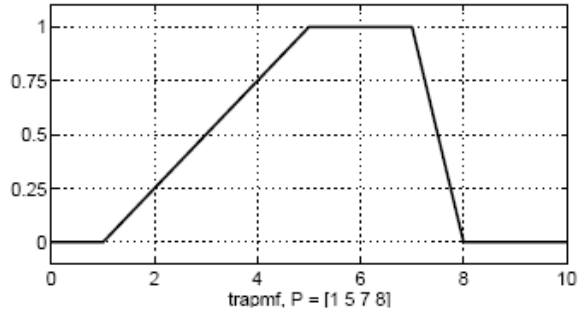
$$\mu_A(x) = 0 \text{ if } x \text{ is not in } A;$$

$$0 < \mu_A(x) < 1 \text{ if } x \text{ is partly in } A.$$

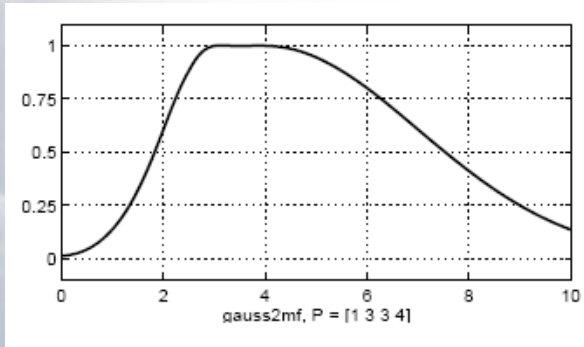
Well known Membership Functions



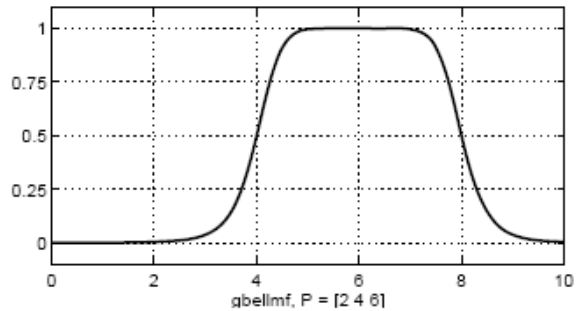
Triangular



Trapezoidal



Gaussian



Bell

Fuzzy Vs Probability

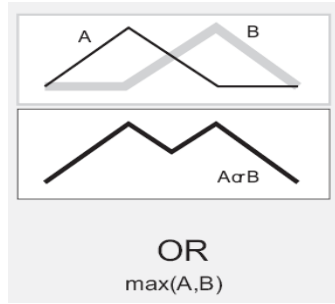
- Fuzzy ≠ Probability => $\mu_A(x) \neq p_A(x)$
- Both map x to a value in [0,1].
- $P_A(x)$ measures our knowledge or ignorance of the truth of the event that x belongs to the set A.
  - Probability deals with uncertainty and likelihood.
- μA(x) measures the degree of belongingness of x to set A and there is no interest regarding the uncertainty behind the outcome of the event x. Event x has occurred and we are interested in only making observations regarding the degree to which x belongs to A.
  - Fuzzy logic deals with ambiguity and vagueness.

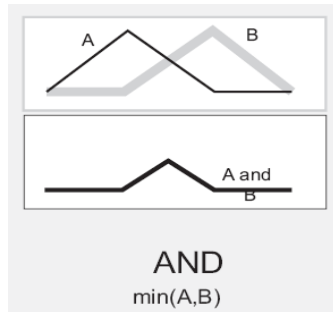
Example

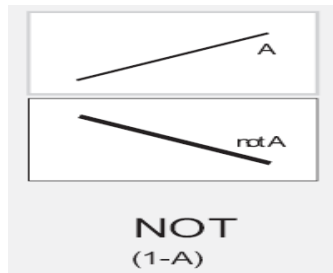
- A bottle of water
- 50% probability of being poisonous means 50% chance.
  - 50% water is clean.
  - 50% water is poisonous.
- 50% fuzzy membership of poisonous means that the water has poison.
  - Water is half poisonous.

Fuzzy Logic Operations

- Fuzzy union operation or fuzzy *OR*
  - $\mu_{A+B} = max[\mu_A(x), \mu_B(x)]$
- Fuzzy intersection operation or fuzzy And
  - $\mu_{A.B} = min[\mu_A(x), \mu_B(x)]$
- Complement operation
  - $\mu_A = 1-\mu_A(x)$





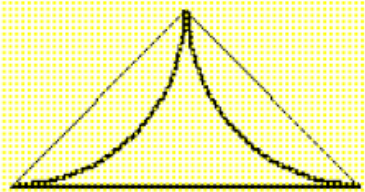
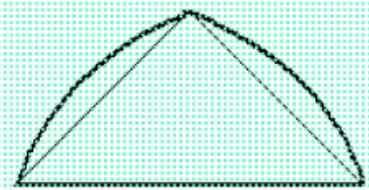


Linguistic variables and hedges

- Wind is *a little* strong.
- Weather is *quite* cold.
- Height is *almost* tall.
- Weight is *very* high.
- Wind, Weather, Height and Weight are linguistic variables.
- A little, Quite, Almost, Very are hedges.
- Strong, Cold, Tall and high are linguistic value.

Example

- Membership of body fitness

Hedge	*Mathematical Expression*	*Graphical Representation*
Very very	$[\mu_A(x)]^4$	
More or less	$\sqrt{\mu_A(x)}$	

Fuzzy Inference

- Fuzzy inference is the process of formulating the mapping from a given input to an output using fuzzy logic.
- If then rules
  - *if temperature is cold then hot water valve is open and cold water valve is shut*
- Rule Base
  - If the distance to intersection (dti) is *far and the speed slow apply gentle breaks*
  - If dti is *near and the speed slow apply medium breaks*
  - If dti is far *and speed fast apply medium breaks*
  - If dti is *near and speed fast apply high breaks*

Fuzzy Inference

- Assume we want to evaluate the health of a person based on his height and weight.
- The input variables are the crisp numbers of the person's height and weight.
- Output is percentage of healthiness.

Step 1:Fuzzification

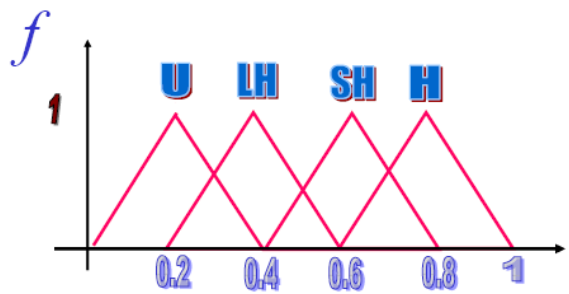
- Fuzzification is a process by which the numbers are changes into linguistic words.

Step 2: Rules

- Rules reflect experts decisions.
- Rules are tabulated as fuzzy words
- Rules can be grouped in subsets
- Rules can be redundant
- Rules can be adjusted to match desired

Rules(Cont.)

- Rules are tabulated as fuzzy words
  - – Healthy (H)
  - – Somewhat healthy (SH)
  - – Less Healthy (LH)
  - – Unhealthy (U)
  - Rule function *f*
- $f = \{U, LH, SH, H\}$



$f = \{U, LH, SH, H\}$

Fuzzy Rule Table

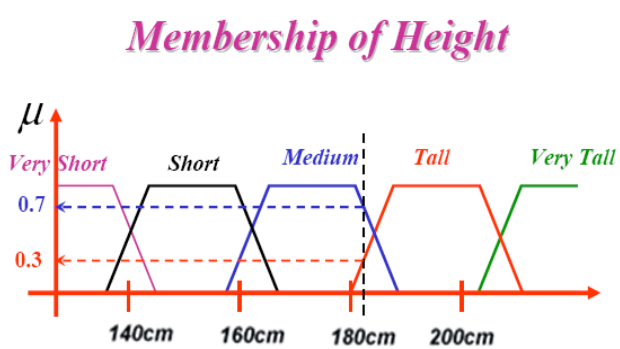
		Weight				
Height		Very Slim	Slim	Medium	Heavy	Very Heavy
	Very Short	H	SH	LH	U	U
	Short	SH	H	SH	LH	U
	Medium	LH	H	H	LH	U
	Tall	U	SH	H	SH	U
	Very Tall	U	LH	H	SH	LH

Step 3:Calculation

- For a given person, compute the membership of his/her weight and height
- Assume that a person height is 185cm
- Assume that the person's weight is 49

$$\mu_{Weight} = \{\mu_{VS} \quad \mu_S \quad \mu_M \quad \mu_H \quad \mu_{VH}\}$$

$$\mu_{Weight} = \{0.8 \quad 0.2 \quad 0 \quad 0 \quad 0\}$$



$$\mu_{height} = \{\mu_{VS} \quad \mu_S \quad \mu_M \quad \mu_T \quad \mu_{VT}\}$$

$$\mu_{height} = \{0 \quad 0 \quad 0.7 \quad 0.3 \quad 0\}$$

Calculation(cont.)

- Rule Activation

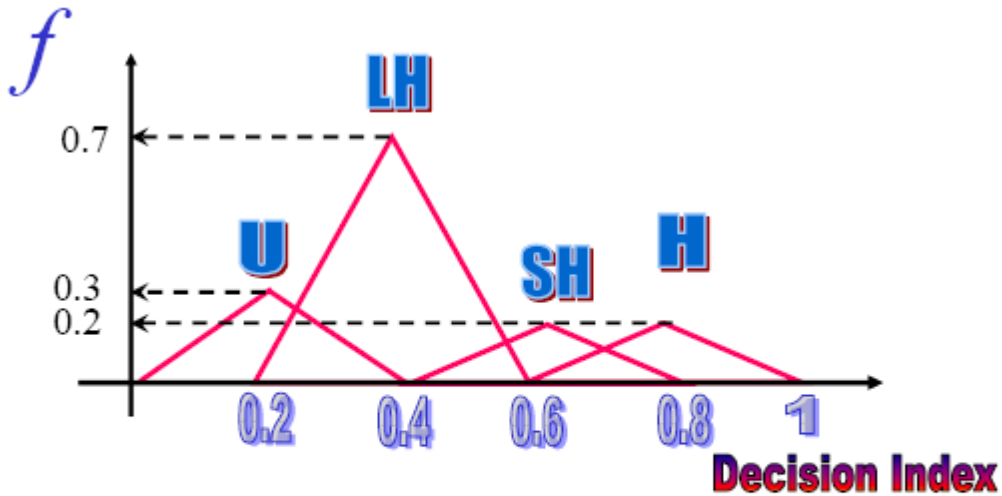
Height \ Weight		0.8	0.2	Medium	Heavy	Very Heavy
	Very Short	H	SH	LH	U	U
	Short	SH	H	SH	LH	U
	0.7	LH	H	H	LH	U
	0.3	U	SH	H	SH	U
	Very Tall	U	LH	H	SH	LH

- Min Operation

Height \ Weight		0.8	0.2	Medium (0)	Heavy (0)	V.Heavy (0)
	V. Short (0)	0	0	0	0	0
	Short (0)	0	0	0	0	0
	0.7	*0.7*	*0.2*	0	0	0
	0.3	*0.3*	*0.2*	0	0	0
	V. Tall (0)	0	0	0	0	0

Calculation(cont.)

- Scaled Fuzzified Decision

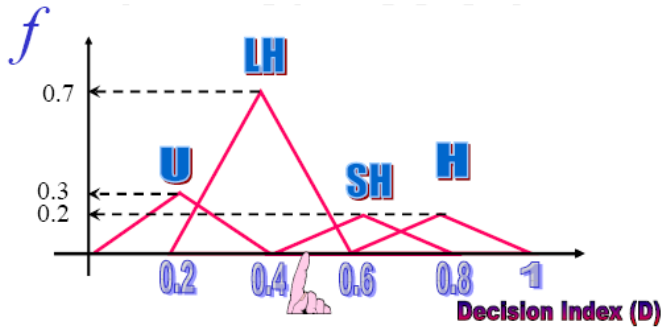


$$f = \{U, LH, SH, H\}$$

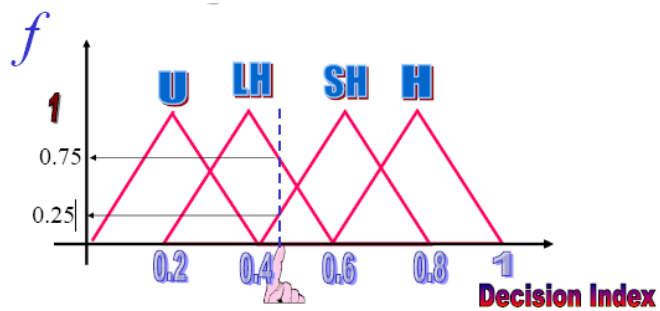
$$f = \{0.3, 0.7, 0.2, 0.2\}$$

Step 4: Final Decision

- Defuzzification



Crisp Decision Index (D) is the centroid
D = 0.4429



Fuzzy Decision Index (D)
75% in Less Healthy group
25% in Somewhat Healthy group

Why Fuzzy Logic?

- Fuzzy logic is conceptually easy to understand.
- Fuzzy logic is flexible.
- Fuzzy logic is tolerant of imprecise data.
- Fuzzy logic can model nonlinear functions of arbitrary complexity.
- Fuzzy logic can be built on top of the experience of experts.
- Fuzzy logic is based on natural language.
- Fuzzy logic can be blended with conventional control techniques.

THANK YOU

Questions?